Int Rev Econ (2014) 61:231–252
DOI 10.1007/s12232-014-0196-6

RESEARCH ARTICLE

# Endogenous population with human and physical capital accumulation

**Wei-Bin Zhang**

Received: 25 January 2013 / Accepted: 5 March 2014 / Published online: 19 March 2014


**Abstract** This paper proposes an economic growth model with population growth and physical and human capital accumulation. The physical capital accumulation is built on the Solow growth model (Solow in Q J Econ 70:65–94, 1956). The education and human capital accumulation is influenced by the Uzawa–Lucas model (Uzawa in Int Econ Rev 6:18–31, 1965; Lucas in J Monet Econ 22:3–42, 1988). The population dynamics are influenced by the Haavelmo population model (Haavelmo in a study in the theory of economic evolution. Haavelmo, Amsterdam, 1954) and the Barro–Becker fertility choice model (Barro and Becker in Econometrica 57:481–501, 1989). We synthesize these dynamic forces in a compact framework, applying an alternative utility function proposed by Zhang (Econ Lett 42:105–110, 1993). The model describes a dynamic interdependence between population change, wealth accumulation, human capital accumulation, and division of labor. We simulate the model to demonstrate the existence of equilibrium points and to plot the motion of the dynamic system. We also examine the effects of changes in the propensity to have children, the mortality rate parameter, the propensity to receive education, the human capital utilization efficiency, and the mortality rate elasticity of human capital upon dynamic paths of the system.

**Keywords** Propensity to have children · Mortality rate · Birth rate · Propensity to receive education

**JEL Classification** O41 · I29 · J10

W.-B. Zhang (✉)
Ritsumeikan Asia Pacific University, 1-1 Jumonjibaru, Beppu-Shi, Oita-ken 874-8577, Japan
e-mail: wbz1@apu.ac.jp

## 1 Introduction

Modern economies are characterized of fast capital accumulation, widely spread education and fast accumulated human capital, and unprecedented population dynamics (such as aging and declining fertility rates in developed economies). In many parts of the world, life expectancy has increased dramatically. Moreover, the both portions of lifetime devoted to education and retirement have increased. To explain the economic mechanisms and dynamic phenomena of these changes, this study proposes a comprehensive analytical framework to examine the dynamic interactions among wealth accumulation, human capital accumulation, and population dynamics with endogenous birth rate and mortality rate.

In order to understand the economic mechanisms of modern economic development, it is necessary to take account of endogenous physical and human capital accumulation as the two factors are significant determinants of modern economic growth. It is well known that the neoclassical growth theory based on the Solow growth model is mainly concerned with endogenous physical capital, even though some extensions of the theory include endogenous human capital.[1] This study follows the traditional neoclassical growth theory in modeling economic production and physical capital accumulation, even though we introduce an alternative approach to determining behavior of households. It is not proper to consider physical capital as a single important factor in explaining growth because human capital is also generally considered as a key determinant of economic growth.[2] In modern times, education is an important way of accumulating human capital. It is well known that the study by Lucas (1988) has caused a great interest among economists in formal modeling of education and economic growth, even though the growth model with education was proposed much earlier, for instance, by Uzawa (1965). Over years, the Uzawa–Lucas model has been extended and generalized in various directions.[3] In this model, both human capital and physical capital are endogenously determined with microeconomic foundations. Nevertheless, only a few studies deal with human and physical accumulation with endogenous population with microeconomic foundations within comprehensive analytical frameworks. The main purpose of our study was to introduce endogenous population into the Uzawa–Lucas model.

The population change consists of dynamics of birth and death. Many factors may interact with changes in fertility. In the literature of population and economic growth, these factors include, for instance, changes in gender gap in wages (Galor and Weil 1996), labor market frictions (Adsera 2005), and age structure (Hock and Weil 2012). Barro and Becker (1989) propose an economic growth model with endogenous fertility in an overlapping generation model. Recently, Bosi and Seegmuller (2012) extend the model by taking account of the heterogeneity of

1 Solow (1956), Burmeister and Dobell (1970), Azariadis (1993), and Barro and Sala-i-Martin (1995).

2 For instance, Hanushek and Kimko (2000), Barro (2001), Lindahl and Krueger (2001), and Castelló-Climent and Hidalgo-Cabrillana (2012).

3 For instance, Jones et al. (1993), Stokey and Rebelo (1995), de la Croix and Licandro (1999), Mino (1996, 2001), Zhang (2003), Lagerlof (2003), Alonso-Carrera and Freire-Sere (2004), Galor (2005), De Hek (2005), Chakraborty and Gupta (2009), and Sano and Tomoda (2010).

households in terms of capital endowments, mortality, and costs per surviving child. The model is built on the quantity-quality trade-off of having children, summarized by the adjustment of the average rearing cost of a surviving child. They show that a rise in mortality increases the time cost per surviving child and enhances economic growth, while reducing demographic growth. As far as this study is concerned, our model has similar concerns to the Schumpeterian growth model with endogenous fertility and human capital accumulation proposed by Chu et al. (2013). R&D-based innovation and human capital accumulation are the two engines of long-run economic growth. The fertility has a negative relationship with the growth rate of human capital. Due to the higher fertility rate's crowding-out effect on household's time endowment, human capital tends to fall. Also, a higher fertility rate has a diluting effect on human capital per member of households. The household chooses the fertility rate by trading off the marginal utility of higher fertility against costs arising from the foregone wages, the dilution of financial assets per capita, as well as the dilution of human capital per capita. Becker et al. (1990) argue that individuals face a quality-quantity trade-off on children. Bringing up children to adulthood and providing them education are costly. In their analysis, growth is the outcome of human capital accumulation only. Physical accumulation is not taken into account. In the studies by Galor and Weil (1999) and Doepke (2004), the quality-quantity trade-off on children has been treated as a factor which affects the transition of economies from a stage of stagnation to perpetual growth.

As argued by Robinson and Srinivasan (1997), there are close relations between economic development and mortality rate. Lancia and Prarolo (2012) develop a politico-economic theory which models processes of interactions between the longevity of life and economic development. In the three-period overlapping generation model, the agents' decisions embrace two dimensions: a private choice about education and a public one on innovation policy. There is an increasing number of economic growth models with endogenous longevity (e.g., Blackburn and Cipriani 2002; Chakraborty 2004; Hazan and Zoabi 2006; Bhattacharya and Qiao 2007). As mentioned by Balestra and Dottori (2012), in this body of the literature, the main and often unique determinant of longevity is represented by health expenditure, either privately or publicly funded. But many studies demonstrate that life expectancy is increased with the aggregate human capital level (e.g., Blackburn and Cipriani 2002; Boucekkine et al. 2002; Cervellati and Sunde 2005). Schultz (1993, 1998) demonstrates that children's life expectancy increases with parent's human capital and education. Using the historical data for 18 countries over the period 1820–2005, Azomahou et al. (2009) demonstrate that the relationship between life expectancy and GDP growth may even be of a S-shaped form. Building an overlapping generations growth model with public health investments, which affect the supply of efficient labor of the old-aged, Fanti and Gori (2011) provide necessary and sufficient conditions for the emergence of endogenous deterministic complex cycles when individuals are perfectly foresighted. They specially demonstrate that the higher the degree of thriftiness, the likelier an economy is exposed to endogenous fluctuations because the need to save when young to support consumption when old is reduced.

Balestra and Dottori (2012) build a growth model of endogenous longevity in a general equilibrium OLG model with public supplies of health care and environment protection. There are also many studies on the impact of old-age dependency on fertility through the pension system (Cigno and Rosati 1996; Wigger 1999). Hock and Weil (2012) propose a model of the interdependence of fertility and the population age structure arising from economic dependency in the presence of intergenerational transfers. They show a possible dynamics that rising old-age dependency decreases the disposable income of the working population, resulting in lower fertility and further population aging. Constructing a model combining the neoclassical growth model of Solow with a comprehensive representation of population dynamics, Fanti et al. (2013) confirm that the age structure is a key dynamic determinant of economic growth. The model shows that proper inclusion of age structure implies the existence of multiple equilibria, allowing up to five states of balanced growth. There are many theoretical models on longevity and human capital and growth (e.g., Boucekkine et al. 2002; Kalemli-Ozcan et al. 2000; Echevarria and Iza 2006; Heijdra and Romp 2008; Ludwig and Vogel 2009; Lee and Mason 2010; and Ludwig et al. 2012). This study is influenced by these traditional models. A unique contribution of this paper is to model population growth in a framework of growth with endogenous human and physical capital accumulation with an alternative approach to the behavior of households. The paper analyzes the link among education, saving, physical and human capital accumulation, and population. The physical capital accumulation is built on the Solow growth model. The education and human capital accumulation are influenced by the Uzawa–Lucas model. The population dynamics are influenced by the Haavelmo population model and the Barro–Becker fertility choice model. We synthesize these dynamic forces in a compact framework, applying an alternative utility function proposed by Zhang (1993). The paper is organized as follows. Section 2 introduces the basic model with wealth accumulation and human capital accumulation with government subsidy on education. Section 3 simulates the model. Section 4 carries out comparative dynamic analysis with regard to some parameters. Section 5 concludes the study.

## 2 The basic model

The economy has one production sector and one education sector. Most aspects of the production sector are similar to the standard one-sector growth model.[4] There is only one (durable) good in the economy. Households own assets of the economy and distribute their incomes to consumption, education, child bearing, and wealth accumulation. The production sectors or firms use physical capital and labor as inputs. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to the factors of production. We assume a homogenous population $N(t)$ at time. Let $T(t)$ and $T_e(t)$ represent for, respectively, the work time and study time of the representative household. The total work time is

[4] See, for instance, Burmeister and Dobell (1970), Azariadis (1993), Barro and Sala-i-Martin (1995), and Zhang (2012).

$T(t)$ $N(t)$. We use $H(t)$ to stand for the level of human capital of the population. The total qualified labor force is

$$\bar{N}(t) = T(t)H^m(t)N(t), \tag{1}$$

where the parameter, $m$, describes how effectively the population uses human capital.

The labor force is distributed between the two sectors. We select the commodity to serve as numerative, with all the other prices being measured relative to its price. We assume that wage rates are identical among all professions. The total capital stock of physical capital, $K(t)$, is allocated between the two sectors. We use $N_e(t)$ and $K_e(t)$ to stand for the labor force and capital stocks employed by the education sector, and $N_i(t)$ and $K_i(t)$ for the labor force and capital stocks employed by the production sector. As labor and capital are assumed fully employed, we have

$$\begin{aligned} K_i(t) + K_e(t) &= K(t) \\ N_i(t) + N_e(t) &= \bar{N}(t). \end{aligned} \tag{2}$$

We rewrite (1) as follows

$$\begin{aligned} n_i(t)k_i(t) + n_e(t)k_e(t) &= k(t) \\ n_i(t) + n_e(t) &= 1, \end{aligned} \tag{3}$$

in which

$$k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad n_j(t), \equiv \frac{N_j(t)}{\bar{N}(t)}, \quad k(t) \equiv \frac{K(t)}{\bar{N}(t)}, \quad j = i, \ e.$$

## 2.1 The industrial sector

The production function is

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad A_i, \ \alpha_i, \ \beta_i > 0, \quad \alpha_i + \beta_i = 1, \tag{4}$$

where $A_i$, $\alpha_i$, and $\beta_i$ are positive parameters. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest, $r(t)$ and wage rate per unit work time (of the qualified labor), $w(t)$, are determined by markets. Hence, for any individual firm, $r(t)$ and $w(t)$ are given at each point in time. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)} = \alpha_i A_i k_i^{-\beta_i}(t), \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)} = \beta_i A_i k_i^{\alpha_i}(t), \tag{5}$$

where $\delta_k$ is the fixed depreciation rate of physical capital.

## 2.2 The education sector

Following Zhang (2012), we assume that the education sector is characterized of perfect competition. The education sector charges students $p(t)$ per unit in time. The education sector pays teachers and capital with the market rates. The cost of the education sector is given by $w(t)N_e(t) + r(t)K_e(t)$. The total education service is measured by the total education time received by the population, $T_e N_0$. The production function of the education sector is a function of $K_e(t)$ and $N_e(t)$

$$F_e(t) = A_e K_e^{\alpha_e}(t)\, N_e^{\beta_e}(t)\,, \quad \alpha_e,\ \beta_e\ >\ 0, \quad \alpha_e + \beta_e = 1\,, \tag{6}$$

where $A_e$, $\alpha_e$, and $\beta_e$ are positive parameters. For given $p(t)$, $r(t)$, and $w(t)$, the marginal conditions are

$$r(t) + \delta_k = \alpha_e A_e p(t) k_e^{-\beta_e}(t),\ w(t) = \beta_e A_e p(t) k_e^{\alpha_e}(t). \tag{7}$$

In this study, we treat education as service. The total time that the population wants to spend in university is the demand for education. The demand for and supply of education balances at any point in time

$$T_e(t)\, N(t) = F_e(t)\,. \tag{8}$$

### 2.3 Human capital dynamics

Following the Uzawa–Lucas model (Uzawa 1965; Lucas 1988), human capital is accumulated through education. We emphasize trade-offs between investment in education and capital accumulation. For simplicity of analysis, we consider that education is privately supported. In the literature of education and economic growth, there are some growth models with public or/and private education.[5] Following Zhang (2014), we propose the following human capital dynamics

$$\dot{H}(t) = \frac{\upsilon_e F_e^{a_e}(t) (H^m(t) T_e(t) N(t))^{b_e}}{H^{\pi_e}(t) N(t)} - \delta_h H(t), \tag{9}$$

where $\delta_h$ (>0) is the depreciation rate of human capital, $\upsilon_e$, $a_e$, and $b_e$, are non-negative parameters. The sign of the parameter, $\pi_e$, is not specified as it may be either negative or positive. The term, $\upsilon_e F_e^{a_e} (H^m T_e N)^{b_e} / H^{\pi_e} N$, describes the contribution to human capital improvement through education. Human capital tends to increase with an increase in the level of education service, $F_e$, and in the (qualified) total study time, $H^m T_e N$. It should be noted that in the literature of education and economic growth, it is assumed that human capital evolves according to the following equation (see Barro and Sala-i-Martin 1995)

$$\dot{H}(t) = H^{\eta}(t) G(T_e(t)),$$

where the function $G$ is increasing as the effort rises with $G(0) = 0$. In the case of $\eta < 1$, there is diminishing return to the human capital accumulation. This formation is due to Lucas (1988). Uzawa's model may be considered a special case of the Lucas model with $\gamma = 0$, $U(c) = c$, and the assumption that the right-hand side of the above equation is linear in the effort. It seems reasonable to consider

[5] Here, we refer to only a few early studies on the issues. Eckstein and Zilcha (1994) propose an overlapping model with compulsory schooling. In Kaganovich and Zilcha (1999) the government allocates tax revenues towards public investment in education and social security benefits. Epple and Romano (1998) and Caucutt (2002) examine school choice with peer effects. In Cardak (2004) and Chen (2005), mixed education systems are presented.

diminishing returns in human capital accumulation: people accumulate it rapidly early in life, then less rapidly, then not at all—as though each additional percentage increment was harder to gain than the preceding one. Solow (2000) adapts the Uzawa formation to the following form $\dot{H}(t) = H(t)\kappa T_e(t)$.

If no effort is devoted to human capital accumulation, then $\dot{H}(0) = 0$ (human capital does not vary as time passes; this results from depreciation of human capital being ignored); if all effort is devoted to human capital accumulation, then $g_H(t) = \kappa$ (human capital grows at its maximum rate; this results from the assumption of potentially unlimited growth of human capital). Between the two extremes, there is no diminishing return to the stock $H(t)$. To achieve a given percentage increase in $H(t)$ requires the same effort. If we consider the above equation as a production for new human capital [i.e., $\dot{H}(t)$], and if the inputs are already accumulated human capital and study time, then this production function is homogenous of degree two. It has strong increasing returns to scale and constant returns to $H(t)$ itself. It can be seen that our approach is more general to the traditional formation with regard to education.

## 2.4 Consumer behaviors

Consumers decide the time of education, consumption level of commodity, number of children, and amount of saving. Different from the optimal growth theory in which utility defined over future consumption streams is used, we use an alternative approach to household proposed by Zhang (1993). To describe behavior of consumers, we denote per capita wealth by $\bar{k}(t)$, where $\bar{k}(t) \equiv K(t)/N(t)$. By the definitions, we have $\bar{k}(t) = k(t)T(t)H^m(t)$.

Per capita current income from the interest payment and the wage payment is

$$y(y) = r(t)\bar{k}(t) + T(t)w(t).$$

We call $y(t)$ the current income in the sense that it comes from consumers' payment for efforts and consumers' current earnings from ownership of wealth. The total value of wealth that consumers can sell to purchase goods and to save is equal to $\bar{k}(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income per head is given by

$$\hat{y}(t) = y(t) + \bar{k}(t).$$

Let $n(t)$ and $p_n(t)$ stand for the birth rate and the cost of birth at time. There are many factors which may affect costs of bringing up children. In this study, for simplicity of analysis, we assume that children will have the same level of wealth as that of the parent. The cost of the parent is thus given by

$$p_n(t) = n(t)\bar{k}(t).$$

Here, we neglect other costs such as time spent on children and purchases of goods and services. It should be noted that in the fertility choice model by Barro and Becker (1989), the cost also includes consumption of goods. Becker (1981) emphasizes costs of the mother's time on rearing children to adulthood. In some societies, women are the primary providers of child care. Wang et al. (1994)

introduce time spent on child bearing into their fertility choice model. The household distributes the total available budget among saving, $s(t)$, consumption of goods, $c(t)$, education, $T_e(t)$, and bearing children, $n(t)$. The budget constraint is

$$c(t) + s(t) + p(t)T_e(t) + \bar{k}(t)n(t) = (1 + r(t))\bar{k}(t) + T(t)w(t). \tag{10}$$

The consumer is faced with the following time constraint

$$T(t) + T_e(t) = T_0, \tag{11}$$

where $T_0$ is the total available time for work and study. Substituting (11) into (10) yields

$$c(t) + s(t) + \bar{p}(t)T_e(t) + \bar{k}(t)n(t) = \bar{y}(t) \equiv (1 + r(t))\bar{k}(t) + T_0 w(t), \tag{12}$$

where $\bar{p}(t) \equiv p(t) + w(t)$ (which is the opportunity cost of education). The right-hand side is the "potential" income that the consumer can obtain by spending all the available time on work. The left-hand side is the sum of the consumption cost, the saving, opportunity cost of education, and the cost of bearing children.

Following Barro and Becker (1989), we assume that the parents' utility is dependent on the number of children. Applying the utility function proposed by Zhang (1993, 2012), we assume that the consumer's utility is dependent on $c(t)$, $s(t)$, $T_e(t)$, and $n(t)$ as follows

$$U(t) = c^{\xi_0}(t) s^{\lambda_0}(t) T_e^{\eta_0}(t) n^{\upsilon_0}(t), \tag{13}$$

where $\xi_0$ is called the propensity to consume, $\lambda_0$ the propensity to own wealth, $\eta_0$ the propensity to obtain education, and $\upsilon_0$ the propensity to have children. Maximizing $U(t)$ subject to (12) yields

$$c(t) = \xi \bar{y}(t), s(t) = \lambda \bar{y}(t), \bar{p}(t)T_e(t) = \eta \bar{y}(t), \bar{k}(t)n(t) = \upsilon \bar{y}(t), \tag{14}$$

where

$$\xi \equiv \rho \xi_0, \quad \lambda \equiv \rho \lambda_0, \quad \eta \equiv \rho \eta_0, \quad \upsilon \equiv \rho \upsilon_0, \quad \rho = \frac{1}{\xi_0 + \lambda_0 + \eta_0 + \upsilon_0}.$$

The demand for education is given by $T_e = \eta \bar{y}/\bar{p}$. The demand for education falls in the price of education and rises in the wealth income. A rise in the propensity to receive education increases the education time when the other variables are fixed. The demand for children is given by $n = \upsilon \bar{y}/\bar{k}$. Demand for children is positively related to the propensity to have children and the wage rate and is negatively related to the wealth level.

## 2.5 The birth and mortality rates and the population dynamics

Before proposing our model, we consider two popular models of population growth. First, we consider the macroeconomic growth model proposed by Haavelmo (1954) [6]

[6] The logistical map has also played an important role in the development of chaos theory. This model is applied to economics by Stutzer (1980), who reforms Haavelmo's population model in continuous form into discrete form.

$$\dot{N}(t) = N(t)\left(a - \frac{\beta N(t)}{Y(t)}\right), \quad a, \beta > 0,$$
$$Y(t) = AN^{\alpha}(t), \quad A > 0, 0 < \alpha < 1,$$

where $N(t)$ is the population, $Y(t)$ is real output, and $a$, $\beta$, $\alpha$, and $A$ are parameters. Substituting $Y(t) = AN^{\alpha}(t)$ into the differential equation yields

$$\frac{\dot{N}(t)}{N(t)} = a - \frac{\beta}{f(t)} = \quad a - \frac{\beta N^{1-\alpha}(t)}{A},$$

where $f(t) \equiv Y(t)/N(t)$ is per capita output. We see that the growth law is a generalization of the familiar logistic form widely used in biological population and economic analysis. If the initial condition satisfies $N(0) > (<)(aA/\beta)^{1/(2-\alpha)}$, then both $N$ and $Y$ will decrease (increase) monotonically until approaching their unique equilibrium points, respectively. In the Haavelmo model, there is neither human capital accumulation nor physical capital accumulation. As the change rate in the population is the birth rate minus the mortality rate, we may interpret that in the Haavelmo model the birth rate ($=a$) is constant and the mortality rate ($=\beta/f(t)$) is negatively related to per capita income.

Another approach is the so-called Ramsey model. According to Chu et al. (2013), the household decides the fertility rate by maximizing the discounted sum of per capita utility across subject to the asset accumulation

$$U = \int_0^{\infty} u(c(t), n(t))e^{-\rho t}dt,$$

s.t.: $\dot{a}(t) = (r(t) - n(t))a(t) + w(t)l(t) - c(t)$, where $c(t)$ is the per capita consumption of final goods at time $t$, $n(t)$ is the number of births per person, $a(t)$ is the amount of financial assets per capita, $r(t)$ is the rate of return on assets, $w(t)$ is the wage rate, and $l(t)$ is human capital-embodied labor supply. Specially, $u = \ln c + \alpha \ln n$, where $\alpha$ is a positive parameter. The total population growth is $\dot{N} = nN$. As the mortality is assumed to be zero in this model, $n$ is also the growth rate of the population. The asset-diluting effect of fertility refers to the phenomenon that an increase in $n(t)$ reduces the amount of assets per capita. The model is similar to Yip and Zhang (1997), which in turn is based on Razin and Ben-Zion (1975).

Being influenced by the above two models, we describe the population dynamics as follows

$$\dot{N}(t) = (n(t) - d(t))N(t), \tag{15}$$

where $n(t)$ and $d(t)$ are, respectively, the birth rate and mortality rate. It should be noted that Tournemaine and Luangaram (2012) construct a model in which technical progress, human capital, and population interact. They examine how individuals' decisions with a trade-off between fertility and education affect the long-run growth rates of technical progress and income per capita growth. In their model, they take account of birth and mortality rates as in (15). But they use the following technology of production of children: $n(t) = bT_b^{\theta}(t)$, where $T_b(t)$ is the time of rearing children and $b$ and $\theta$ are parameters. In their model, the mortality rate is assumed to be constant. Our model introduce endogenous mortality rate. From (14), we see that the birth rate is given by

$$n(t) = \frac{\upsilon \bar{y}(t)}{\bar{k}(t)}. \tag{16}$$

As mentioned before, in the Haavelmo model, the mortality rate is negatively related to per capita income. In this study we assume that the mortality rate is negatively related to the disposable income and the level of human capital in the following way

$$d(t) = \frac{\bar{\upsilon}}{\bar{y}^a(t) H^b(t)}, \tag{17}$$

where $\bar{\upsilon} \geq 0$, $a \geq 0$, and $b \geq 0$. We call $\bar{\upsilon}$ the mortality rate parameter. As in the Haavelmo model, an improvement in living conditions implies that people live longer. But different from the Haavelmo model, we assume that the accumulated human capital negatively affects the mortality rate. Insert (16) and (17) in (15)

$$\dot{N}(t) = \left( \frac{\upsilon \bar{y}(t)}{\bar{k}(t)} - \frac{\bar{\upsilon}}{\bar{y}^a(t) H^b(t)} \right) N(t). \tag{18}$$

It should be noted that to properly describe the population change, we need not only to take account of the dynamics of birth and death rates, but also to model the age structure (for instance, Fanti et al. 2013). Like in most of the models in continuous time in the literature of economic growth and population change, for simplicity, we omit complicated issues related to the age structure.

### 2.6 Wealth dynamics

We now find dynamics of wealth accumulation. According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) = \lambda \bar{y}(t) - \bar{k}(t). \tag{19}$$

We have thus built the dynamic model. We now examine dynamics of the model.

## 3 The dynamics and its properties

The previous section constructed the growth model with endogenous population, physical capital, and human capital. There are three differential equations for population change, wealth accumulation, and human capital dynamics. As the three variables interrelated, we are faced with difficulties of analyzing three-dimensional nonlinear differential equations. It is almost impossible to know analytical properties of the nonlinear dynamic system. Nevertheless, we can rely on computer simulation to follow the motion of the dynamic system. The following lemma shows that we can get a computational procedure to plot the motion of the economic system. We now introduce $z(t) \equiv (r(t) + \delta_k)/w(t)$. We show that the dynamics can be expressed by the three-dimensional differential equations system with $z(t)$, $N(t)$, and $H(t)$ as the variables.

**Lemma** The dynamics of the economic system is governed by the three-dimensional differential equations

$$\begin{aligned} \dot{z}(t) &= \tilde{\Omega}_z(z(t), N(t), H(t)), \\ \dot{N}(t) &= \tilde{\Omega}_N(z(t), N(t), H(t)), \\ \dot{H}(t) &= \tilde{\Omega}_H(z(t), N(t), H(t)), \end{aligned} \tag{20}$$

where $\tilde{\Omega}_z$, $\tilde{\Omega}_N$, and $\tilde{\Omega}_H$ are functions of $z(t)$, $N(t)$, and $H(t)$ defined in the Appendix. Moreover, all the other variables are determined as functions of $z(t)$, $N(t)$, and $H(t)$ at any point in time by the following procedure: $k_i(t) = \tilde{\alpha}_i / z(t) \to k_e(t)$ by (23) $\to p(t)$ by (24) $\to r(t)$ and $w(t)$ by (5) $\to \bar{p}(t) = p(t) + w(t) \to k(t)$ by (33) $\to T(t)$ by (28) $\to T_e(t)$ by (11) $\to \bar{y}(t)$ by (26) $\to c(t),\ s(t)$, and $n(t)$ by (14) $\to n_i(t)$ and $n_e(t)$ by (25) $\to \bar{N}(t)$ by (22) $\to N_i(t) = n_i(t)\bar{N}(t) \to N_e(t) = n_e(t)\bar{N}(t) \to K_i(t) = k_i(t)N_i(t) \to K_e(t) = k_e(t)N_e(t) \to F_i(t)$ by (4) $\to F_e(t)$ by (6).

The differential equations system (20) contains three variables, $z(t)$, $N(t)$, and $H(t)$. As the expressions are too complicated, we simulate the model to illustrate the behavior of the system. In the remainder of this study, we specify the depreciation rates by $\delta_k = 0.05$, $\delta_h = 0.05$, and let $T_0 = 1$. The requirement $T_0 = 1$ will not affect our analysis. The depreciation rate of physical capital is often fixed around 0.05 in economic studies. According to Stokey and Rebelo (1995), it is reasonable to consider the depreciation rate of human capital a range between 0.03 and 0.08 for the US economy. We specify the other parameters as follows

$$\begin{aligned} &\alpha_i = 0.35\,,\ \ \alpha_e = 0.45\,,\ \ \lambda_0 = 0.7\,,\ \ \xi_0 = 0.08,\ \ \eta_0 = 0.01,\ \ \upsilon_0 = 0.2,\ \ A_i = 1.2\,,\ \ A_e = 1.2\,, \\ &m = 0.8\,,\ \ v_e = 1.3\,,\ \ a_e = 0.2,\ \ b_e = 0.1,\ \ \pi_e = -0.1\ \ a = 0.3\ \ b = 0.1\,,\ \ \bar{\upsilon} = 0.6\,. \end{aligned} \tag{21}$$

The propensity to save is 0.7 and the propensity to receive education is 0.01. The propensity to consume goods is 0.08. The technological parameters of the two sectors are specified at $A_i = A_e = 1.2$. The conditions $\pi_e = -0.1$ means that the learning by education exhibits decreasing effects in human capital. The human capital utilization efficiency is 0.8. To follow the motion of the system, we specify initial conditions

$$z(0) = 0.3\,,\quad N(0) = 2.7\,,\quad H(0) = 4\,.$$

The simulation result is plotted in Fig. 1. The population and human capital rise initially and then fall. The birth rate falls. The mortality rate falls initially and then rises. Most of the labor force is employed by the industrial sector. The motion of the rest variables is plotted in Fig. 1.

We observe that the variables tend to become stationary over time. The simulation demonstrates that the dynamic system with the specified parameter

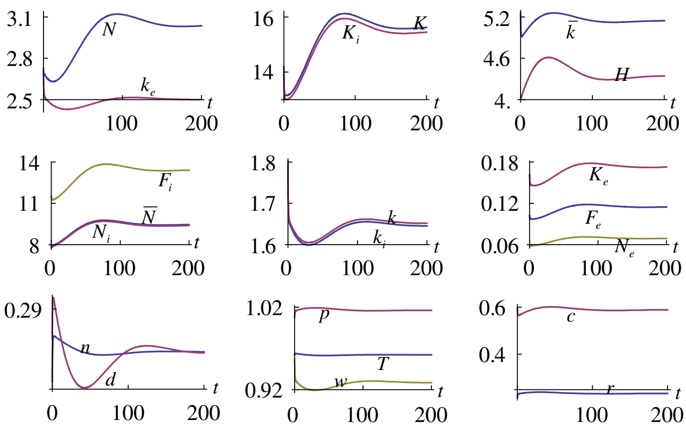


**Fig. 1** Motion of the economic system

values has a unique equilibrium point. We list the equilibrium values of the variables as follows

$$N = 3.04\,,\quad H = 4.34\,,\quad K = 15.65\,,\quad \bar{N} = 9.47\,,\quad N_i = 9.40\,,\quad N_e = 0.067\,,$$

$$K_i = 15.48\,,\quad K_e = 0.17\,,\quad k_i = 1.65\,,\quad k_e = 2.50\,,\quad F_i = 13.43\,,\quad F_e = 0.12\,,\quad n = d = 0.286\,,$$

$$k = 1.65\,,\quad r = 0.234\,,\quad p = 1.02\,,\quad w = 0.93\,,\quad \bar{k} = 5.14\,,\quad T = 0.96\,,\quad c = 0.59\,.$$

We calculate the three eigenvalues: $-0.21$, $-0.08$, and $-0.04$. As the three eigenvalues are real and negative, the unique equilibrium is locally stable. Hence, the system always approaches its equilibrium if it is not far from the equilibrium. As our simulation is specified with the case that neither population nor human capital dynamics exhibits increasing returns, it has a unique stable equilibrium point. It should be noted that if human capital and population are constant, as far as equilibrium and stability are concerned, the model in this study is similar to the standard Solow model (which has a unique stable equilibrium point), rather than the standard Ramsey model (which has a unique saddle point). This point is discussed in Zhang (2005). The Ramsey growth model with endogenous population by Yip and Zhang (1997) has different stability properties from the model proposed in this study. This difference partly results from the two studies take on different utility functions.

## 4 Comparative dynamic analysis in some parameters by simulation

As the system has a unique stable equilibrium point, we can effectively conduct comparative dynamic analysis. We now examine effects of changes in some parameters on dynamic processes of the system. First, we introduce a variable $x(t)$ to stand for the change rate of the variable, $\bar{\Delta}x(t)_e$ in percentage due to changes in a parameter value.

### 4.1 A rise in the mortality rate parameter

We now examine the case that the mortality rate parameter is increased as follows: $\bar{\upsilon}: \ 0.6 \ \Rightarrow \ 0.62$. The simulation results are plotted in Fig. 2. In order to examine how each variable is affected over time, we should follow the motion of the entire system as each variable is related to all the others in the dynamic system. When $\bar{\upsilon}$ is increased, the mortality rate is increased and the population is reduced. As the population falls, the total labor force is reduced. The total capital stocks, labor and capital inputs, and output levels of the two sectors are reduced. The human capital, education time, and consumption level and wealth per person are increased. The education fee and the rate of interest are increased, while the wage rate is reduced. The birth rate is increased initially, but not affected in the long term. The mortality rate rises initially, then falls, but is not affected in the long term. Although the birth and mortality rates are not almost affected by the change in the mortality rate parameter in the long terms, the other micro- and macrovariables are strongly affected.

### 4.2 A rise in the propensity to have children

We allow the propensity to have children to be increased as follows: $\upsilon_0$:0.2 $\Rightarrow$ 0.22. The simulation results are plotted in Fig. 3. When the propensity to have children is increased, the birth rate is increased. The population is increased, while human capital is reduced. The net result of the rise in the population and the reduction in human capital leads to the rise in the total labor supply. The total capital is increased in association with the rise in the population and labor supply. The wage rate falls initially in association with the fall of the capital intensities of the two sectors. As

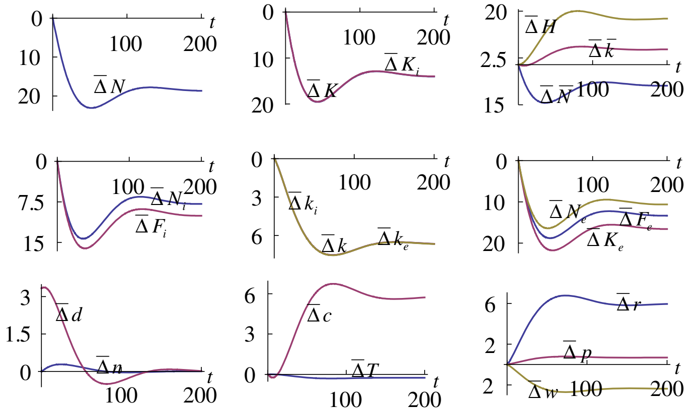


**Fig. 2** Rise in the mortality rate parameter

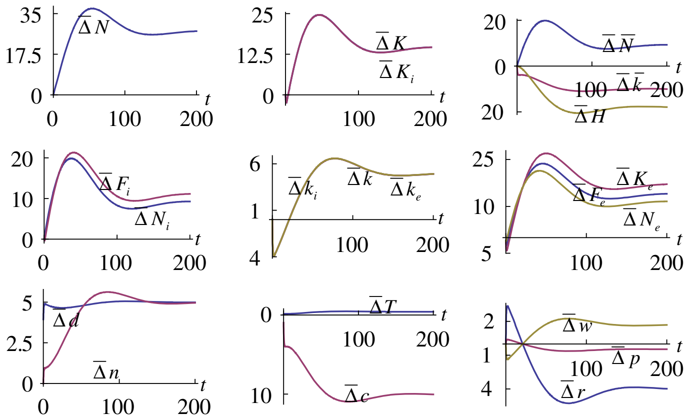


**Fig. 3** Rise in the propensity to have children

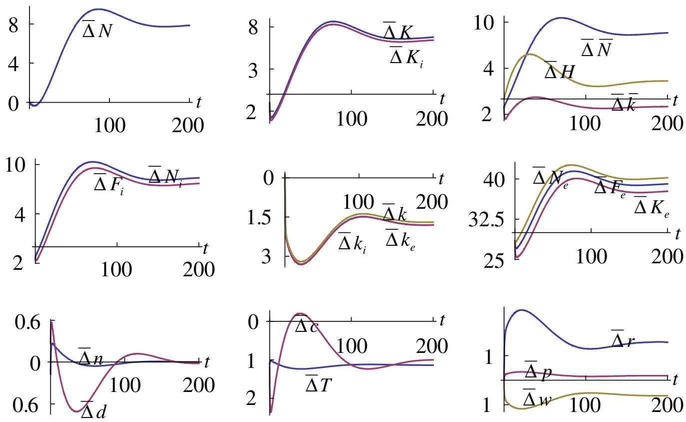


**Fig. 4** Rise in the propensity to receive education

the capital intensities of the two sectors are increased in the long term, the wage rate is increased in the long term. The labor and capital inputs and output levels of the two sectors are increased. The education time is slightly reduced as the work time is increased. The rate of interest and the education price fall. The mortality rate is increased as human capital, and the wealth level per person are reduced.

## 4.3 A rise in the propensity to receive education

We consider the propensity to receive education be increased in the following way: $\eta_0$:0.01 $\Rightarrow$ 0.013. The simulation results are plotted in Fig. 4. As people are more interested in receiving education, they increase education time. As they spend more time and money on education, the education fee is increased and the education sector employs more capital and people and the education sector's output is increased. As people spend more time on formal education, their human capital is increased. The birth and mortality rates rise initially and then fall. The population is increased, while the human capital is reduced. The net result leads to the fall in the capital intensities of the two sectors. The wage rate falls and the rate of interest rises. The labor and capital inputs and output level of the industrial sector are reduced initially, but increased in the long term. The labor and capital inputs and output level of the education sector are increased. As the household's propensity to receive education increases, the per person level of consumption and wealth are reduced in the long term. It should be noted that according to Arrow (1973), a stronger interest in education may not lead to human capital and economic growth. The conclusion results from the assumption that students choose education also for the purpose of signaling. In the literature of education and economics, the signaling view of education was initially formally presented by Spence (1973), Arrow (1973), and Stiglitz (1975). This implies that direct productivity gains are not necessary to explain the choice of quantity and quantity of education. Our result shows that even if we consider that people increase their propensity to receive education and they increase formal education for learning, the long-run consumption and wealth levels per person are reduced. This occurs because we take account of endogenous population growth.

## 4.4 A rise in the human capital utilization efficiency

A person may spend much time on mastering, for instance, the Chinese classical literature. Nevertheless, the time-consumed result may not become useful for economic activities. Education and economic efficiency is closely related, but not necessarily positively. How changes in human capital may affect productivity is through the parameter of human capital utilization efficiency. We now increase the parameter as follows: $m$:0.8 $\Rightarrow$ 0.82. The simulation results are plotted in Fig. 5. As the efficiency is improved, the total labor supply is increased. As the mortality rate

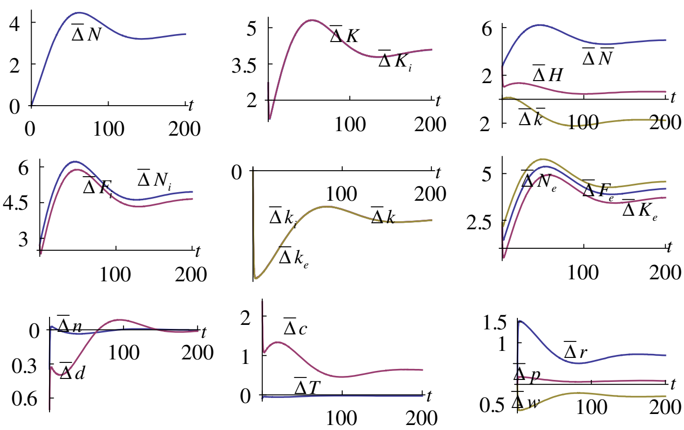

Fig. 5 Rise in the human capital utilization efficiency

initially falls as a consequence of improved human capital and birth rate is slightly affected, the population is increased initially. Late on the population falls as the mortality rate is increased, and the birth rate is slightly affected. As a net result of the fall in per person wealth and the increase in the population, the total capital stocks are increased. The labor and capital inputs and output levels of the two sectors are reduced. As the capital intensities of the two sectors fall, the wage rate falls and the rate of interest rises.

### 4.5 A rise in the mortality rate elasticity of human capital

It is generally held that education and human capital have strong effects in population growth. We now provide some insights into this issue by changing the mortality rate elasticity of human capital rate by $b{:}0.1 \Rightarrow 0.12$. A rise in $b$ initially reduces the mortality rate and increases the population. In association with the rise in population, the total labor supply and total capital are increased. In association with the increases in the total capital and labor force, the labor and capital inputs and output levels of the two sectors are increased. As the capital sock grows faster than the labor supply, the capital intensities of the two sectors are increased. The rises in the capital intensities result in the fall in the rate of interest and the rise in wage rate. The demand for education falls in association of the fall in the mortality rate. The net result of the fall in education time and the rise in the total output of the education sector is that the level of human capital is reduced. As the human capital level falls, the mortality rate rises before it falls to approach zero. In the long term, both the birth rate and mortality rate are not affected by the rise in the mortality rate elasticity of human capital rate, even though the population is increased. As the population is increased and human capital is reduced, we see that the consumption and wealth levels per person are reduced. We also analyze the impact of the following change: $a{:}0.3 \Rightarrow 0.32$. The effects are quantitatively similar to those effects in Fig. 6.

## 5 Concluding remarks

This paper introduced endogenous population growth model into the Uzawa-Lucas two-sector model with Zhang's utility function. The paper models a dynamic interdependence between the population, physical capital, and human capital. We emphasized the role of economic changes and human capital dynamics on the birth and mortality rates. We took account of the learning by education in modeling human capital change. We simulated the model to demonstrate the existence of equilibrium points and plot the motion of the dynamic system. We also examined the effects of changes in the propensity to have children, the mortality rate parameter, the propensity to receive education, the human capital utilization efficiency, and the mortality rate elasticity of human capital. Our simulation results also provide some insights into modern economic development with demographic transition. It has been observed that many economies have experienced the decline of fertility rate in association with economic development (e.g., Kirk 1996; Enrlich

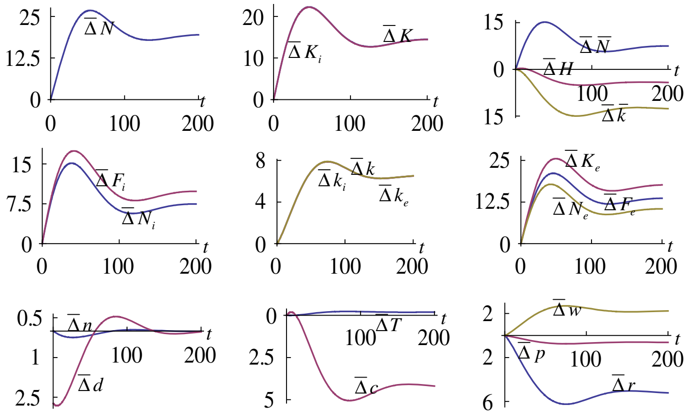


**Fig. 6** Rise in the mortality rate elasticity of human capital

and Lui 1997; Galor 2012; Varvarigos and Zakaria 2013). It is reasonable to expect that in the literature of demographic growth and economic development, there are many determinants for explaining the phenomenon. Our simulation results demonstrate that during transitional processes, the birth rate is negatively affected by, for instance, the propensity to receive education in the late stage, the human capital utilization efficiency in the late stage, and positively affected by the mortality rate parameter, the propensity to receive education in the initial stage, the propensity to have children, human capital utilization efficiency in the initial stage, and the mortality rate elasticity of human capital in the initial stage. Only a few formal growth models can address the relations between the birth rate and the different economic forces in a single framework. We can examine the effects of various forces on the birth rate because our model integrates some key models in the literature of economic growth and the literature of population within a compact framework. Indeed, we acknowledge that although it is useful for the purposes mentioned above, our model is based in a rather eclectic way on different contributes and abstracts from the choice of sound microfoundations. Moreover, it should be noted that our simulation is conducted with limited set of parameter values, even though it is straightforward to simulate any set of parameters using the computational procedure given in the paper. We may observe other qualitative properties of the dynamic system if the parameters are taken on other values.

**Acknowledgments** The author is grateful for the constructive comments and suggestions of Editor Pierluigi Porta and the three anonymous referees. He is also grateful for the financial support from the Grants-in-Aid for Scientific Research (C), Project No. 25380246, Japan Society for the Promotion of Science.

## Appendix: Proving the Lemma

We now show that the dynamics can be expressed by a three-dimensional differential equations system. From (5) and (7), we obtain

$$z \equiv \frac{r+\delta_k}{w} = \frac{\tilde{\alpha}_i}{k_i} = \frac{\tilde{\alpha}_e}{k_e}, \tag{22}$$

where $\tilde{\alpha}_j \equiv \alpha_j/\beta_j$. From (22) we have

$$k_e = \alpha\, k_i, \tag{23}$$

where $\alpha \equiv \alpha_e\beta_i/\alpha_i\beta_e$ ($\neq 1$ assumed). From (5), we determine $r$ and $w$ as functions of $k_i$. From (23), (5) and (7), we obtain

$$p = \beta_0\, k_i^{\beta}, \tag{24}$$

where

$$\beta_0 \equiv \frac{\alpha^{\beta_e}\,\alpha_i\,A_i}{\alpha_e\,A_e}, \quad \beta \equiv \beta_e - \beta_i.$$

We determine $p$ as a function of $k_i$. As $k_i = \tilde{\alpha}_i/z$, we determine $k_i$, $k_e$, $p$, $r$, $w$, and $\bar{p}$ as functions of $z$.

From (22) and (1), we solve the labor distribution as functions of $k_i$ and $k$

$$n_i = \frac{\alpha\,k_i - k}{\bar{\alpha}\,k_i}, \quad n_e = \frac{k - k_i}{\bar{\alpha}\,k_i}, \tag{25}$$

where $\bar{\alpha} \equiv \alpha - 1$. Insert (2) and $\bar{k} = k\,T\,H^m$ in the definition of $\bar{y}$ in (12)

$$\bar{y} = (1+r)\,k\,T\,H^m + T_0\,w. \tag{26}$$

From $\bar{p}\,T_e = \eta\,\bar{y}$ in (14) and (26), we have

$$\bar{p}\,T_e = (1+r)\,\eta\,k\,T\,H^m + \eta\,T_0\,w. \tag{27}$$

From (11) and (27), we have

$$T = \frac{(\bar{p} - \eta\,w)\,T_0}{(1+r)\,\eta\,k\,H^m + \bar{p}}. \tag{28}$$

Insert (28) in (1)

$$\bar{N}(k,\ z,\ N,\ H) = \frac{(\bar{p} - \eta\,w)\,H^m\,N\,T_0}{(1+r)\,\eta\,k\,H^m + \bar{p}}. \tag{29}$$

From (8) and (6), we have

$$T_e = A_e\,T\,n_e\,H^m\,k_e^{\alpha_e}. \tag{30}$$

From (30) and (11), we have

$$T = \frac{T_0}{1 + A_e\,n_e\,H^m\,k_e^{\alpha_e}}. \tag{31}$$

From (28) and (31), we solve

$$n_e = \left(\frac{(1+r)\eta k H^m \bar{p}}{\bar{p} - \eta w} - 1\right)\frac{1}{A_e\,H^m\,k_e^{\alpha_e}}. \tag{32}$$

From (25) and (32), we solve

$$k(z\,,\ N,\ H) = \left(\frac{A_e\, H^m\, k_e^{\alpha_e}}{\bar{\alpha}} + \frac{\eta\, w}{\bar{p} - \eta\, w}\right)\left[\frac{A_e\, k_e^{\alpha_e}}{\bar{\alpha}\, k_i} - \frac{(1+r)\,\eta}{\bar{p} - \eta\, w}\right]^{-1} H^{-m}. \tag{33}$$

We determine all the variables as functions of $z(t)$, $N(t)$, and $H(t)$ at any point in time by the following procedure: $k_i = \tilde{\alpha}_i/z$ by (22) $\rightarrow$ $k_e$ by (23) $\rightarrow$ $p$ by (24) $\rightarrow$ $r$ and $w$ by (5) $\rightarrow$ $\bar{p} = p + w$ $\rightarrow$ $k$ by (33) $\rightarrow$ $T$ by (28) $\rightarrow$ $T_e$ by (11) $\rightarrow$ $\bar{y}$ by (26) $\rightarrow$ $c$, $s$, and $n$ by (14) $\rightarrow$ $n_i$ and $n_e$ by (25) $\rightarrow$ $\bar{N}$ by (22) $\rightarrow$ $N_i = n_i \bar{N}$ $\rightarrow$ $N_e = n_e \bar{N}$ $\rightarrow$ $K_i = k_i N_i$ $\rightarrow$ $K_e = k_e N_e$ $\rightarrow$ $F_i$ by (4) $\rightarrow$ $F_e$ by (6).

From this procedure, (9) and (18), it is straightforward to show that the motion of human capital and the population can be expressed as function of $z(t)$, $N(t)$, and $H(t)$ at any point in time

$$\dot{H} = \tilde{\Omega}_H(z\,,\ N\,,\ H)\,,$$
$$\dot{N}(t) = \tilde{\Omega}_N(z\,,\ N\,,\ H)\,. \tag{34}$$

We now show that change in $z(t)$ can also be expressed as a differential equation in terms of $z(t)$, $N(t)$, and $H(t)$. From (19), we have

$$\dot{\bar{k}} = \tilde{\Omega}_0(z\,,\ N\,,\ H)\ \equiv\ \lambda \bar{y} - \bar{k}\,. \tag{35}$$

Taking derivatives of $\bar{k} = k\,T\,H^m$ with respect to time, we have

$$\frac{\dot{\bar{k}}}{\bar{k}} = \left(\frac{1}{k}\frac{\partial k}{\partial z} + \frac{1}{T}\frac{\partial T}{\partial z}\right)\dot{z} + \left(\frac{1}{k}\frac{\partial k}{\partial N} + \frac{1}{T}\frac{\partial T}{\partial N}\right)\tilde{\Omega}_N + \left(\frac{1}{k}\frac{\partial k}{\partial H} + \frac{1}{T}\frac{\partial T}{\partial H} + \frac{m}{H}\right)\tilde{\Omega}_H\,, \tag{36}$$

where we also use (34). From (35) and (36), we solve

$$\dot{z} = \tilde{\Omega}_z(z\,,\ N\,,\ H)$$
$$\equiv \left[\frac{\tilde{\Omega}_0}{\bar{k}} - \left(\frac{1}{k}\frac{\partial k}{\partial N} + \frac{1}{T}\frac{\partial T}{\partial N}\right)\tilde{\Omega}_N - \left(\frac{1}{k}\frac{\partial k}{\partial H} + \frac{1}{T}\frac{\partial T}{\partial H} + \frac{m}{H}\right)\tilde{\Omega}_H\right]\left(\frac{1}{k}\frac{\partial k}{\partial z} + \frac{1}{T}\frac{\partial T}{\partial z}\right)^{-1}. \tag{37}$$

The three differential equations (34) and (37) contain three variables $z(t)$, $N(t)$, and $H(t)$. We thus proved the lemma.

## References


Adsera A (2005) Vanishing children: from high unemployment to low fertility in developed countries. Am Econ Rev 95:189–193

Alonso-Carrera J, Freire-Sere MJ (2004) Multiple equilibria, fiscal policy and human capital accumulation. J Econ Dyn Control 28:841–856

Arrow K (1973) Higher education as a filter. J Public Econ 2:193–216

Azariadis C (1993) Intertemporal macroeconomics. Blackwell, Oxford

Azomahou TT, Boucekkine R, Diene B (2009) A closer look at the relationship between life expectancy and economic growth. Int J Econ Theory 5:201–244

Balestra C, Dottori D (2012) Aging society, health and the environment. J Popul Econ 25:1045–1076

Barro RJ (2001) Human capital and growth. Am Econ Rev, Papers and Proceedings 91:12–17

Barro RJ, Becker GS (1989) Fertility choice in a model of economic growth. Econometrica 57:481–501
Barro RJ, Sala-i-Martin X (1995) Economic growth. McGraw-Hill Inc, New York
Becker GS (1981) A treatise on the family. Harvard University Press, Cambridge, MA
Becker GS, Murphy KM, Tamura R (1990) Human capital, fertility, and economic growth. J Polit Econ 98:12–37
Bhattacharya J, Qiao X (2007) Public and private expenditures on health in a growth model. J Econ Dyn Control 31:2519–2535
Blackburn K, Cipriani GP (2002) A model of longevity, fertility and growth. J Econ Dyn Control 26:187–204
Bosi S, Seegmuller T (2012) Mortality differential and growth: what do we learn from the Barro–Becker Model? Math Popul Stud 19:27–50
Boucekkine R, de la Croix D, Licandro O (2002) Vintage human capital. demographic trends, and endogenous growth. J Econ Growth 104:340–375
Burmeister E, Dobell AR (1970) Mathematical theories of economic growth. Collier Macmillan, London
Cardak BA (2004) Ability, education, and income inequality. J Public Econ Theory 6:239–276
Castelló-Climent A, Hidalgo-Cabrillana A (2012) The role of educational quality and quantity in the process of economic development. Econ Educ Rev 31:391–409
Caucutt EM (2002) Educational vouchers when there are peer group effects—size matters. Int Econ Rev 43:195–222
Cervellati M, Sunde U (2005) Human capital formation, life expectancy, and the process of development. Am Econ Rev 95:1653–1672
Chakraborty S (2004) Endogenous lifetime and economic growth. J Econ Theory 116:119–137
Chakraborty B, Gupta MR (2009) Human capital, inequality, endogenous growth and education subsidy: a theoretical analysis. Res Econ 63:77–90
Chen HJ (2005) Educational systems, growth and income distribution: a Quantitative Study. J Dev Econ 76:325–353
Chu AC, Cozzi G, Liao CH (2013) Endogenous fertility and human capital in a Schumpeterian Growth Model. J Popul Econ 26:181–202
Cigno A, Rosati FC (1996) Jointly determined saving and fertility behaviour: theory and estimates for Germany, Italy, UK and USA. Eur Econ Rev 40:1561–1589
De Hek PA (2005) On taxation in a two-sector endogenous growth model with endogenous labor supply. J Econ Dyn Control 30:655–685
De la Croix D, Licandro O (1999) Life expectancy and endogenous growth. Econ Lett 65:255–263
Doepke M (2004) Accounting for fertility decline during the transition to growth. J Econ Growth 9:347–383
Echevarria CA, Iza A (2006) Life expectancy, human capital, social security and growth. J Public Econ 90:2324–2349
Eckstein Z, Zilcha I (1994) The effects of compulsory schooling on growth, income distribution and welfare. J Public Econ 54:339–359
Enrlich I, Lui F (1997) The problem of population and growth: a review of the literature from malthus to contemporary models of endogenous population and endogenous growth. J Econ Dyn Control 21:205–242
Epple D, Romano RE (1998) Competition between public and private schools, vouchers and peer-group effects. Am Econ Rev 88:33–62
Fanti L, Gori L (2011) Public health spending, old-age productivity and economic growth: chaotic cycles under perfect foresight. J Econ Behav Organ 78:137–151
Fanti L, Iannelli M, Manfredi P (2013) Neoclassical growth with endogenous age distribution: poverty vs low-fertility traps as steady states of demographic transitions. J Popul Econ 26:1457–1484
Galor O (2005) From stagnation to growth: unified growth theory. In: Aghion P, Durlauf SN (eds) Handbook of economic growth. Elsevier, Amsterdam, North Holland
Galor O (2012) The demographic transition: cause and consequences. Cliometrica 6:1–28
Galor O, Weil DN (1996) The gender gap, fertility, and growth. Am Econ Rev 86:374–387
Galor O, Weil D (1999) From malthusian stagnation to modern growth. Am Econ Rev 89:150–154
Haavelmo T (1954) A study in the theory of economic evolution. Haavelmo, Amsterdam, North-Holland
Hanushek E, Kimko D (2000) Schooling, labor-force quality and the growth of nations. Am Econ Rev 90:1194–1204
Hazan M, Zoabi H (2006) Does longevity cause growth? a theoretical critique. J Econ Growth 11:363–376

Heijdra BJ, Romp WE (2008) A life-cycle overlapping-generations model of the small open economy. Oxford Econ Pap 60:88–121
Hock H, Weil DN (2012) On the dynamics of the age structure, dependency, and consumption. J Popul Econ 25:1019–1043
Jones LE, Manuelli RE, Rossi PE (1993) Optimal taxation in models of endogenous growth. J Polit Econ 101:485–517
Kaganovich M, Zilcha I (1999) Education, social security and growth. J Public Econ 71:289–309
Kalemli-Ozcan S, Ryder HE, Weil DN (2000) Mortality decline, human capital investment, and economic growth. J Dev Econ 62:1–3
Kirk D (1996) Demographic transition theory. Popul Stud 50:361–387
Krueger AB (1999) Experimental estimates of education production functions. Q J Econ 114:497–532
Lagerlof NP (2003) From malthus to modern growth: the three regimes revisited. Int Rev Econ 44:755–777
Lancia F, Prarolo G (2012) A politico-economic model of aging, technology adoption and growth. J Popul Econ 25:989–1018
Lee R, Mason A (2010) Fertility, human capital, and economic growth over the demographic transition. Eur J Popul 26:159–182
Lindahl M, Krueger AB (2001) Education for growth: why and for whom? J Econ Lit 39:1101–1136
Lucas RE (1988) On the mechanics of economic development. J Monet Econ 22:3–42
Ludwig A, Vogel E (2009) Mortality, fertility, education and capital accumulation in a simple OLG economy. J Popul Econ 23:703–735
Ludwig A, Schelkle T, Vogel E (2012) Demographic change, human capital and welfare. Rev Econ Dyn 15:94–107
Mino K (1996) Analysis of a two-sector model of endogenous growth with capital income taxation. Int Econ Rev 37:227–251
Mino K (2001) Optimal taxation in dynamic economies with increasing returns. Jpn World Econ 13:235–253
Razin A, Ben-Zion U (1975) An intergenerational model of population growth. Am Econ Rev 65:923–933
Robinson JA, Srinivasan TN (1997) Long-term consequence of population growth: technological change, natural resources, and the environment. In: Rozenzweig MR, Stark O (eds) Handbook of population and family economics. Elsevier, Amsterdam, North-Holland
Sano K, Tomoda Y (2010) Optimal public education policy in a two sector model. Econ Model 27:991–995
Schultz PT (1993) Mortality decline in the low-income world: causes and consequences. Am Econ Rev 83:337–342
Schultz PT (1998) Health and schooling investments in Africa. J Econ Perspect 13:67–88
Solow R (1956) A contribution to the theory of growth. Q J Econ 70:65–94
Solow R (2000) Growth theory: an exposition. Oxford University Press, New York
Spence M (1973) Job market signaling. Q J Econ 87:355–374
Stiglitz JE (1975) The theory of screening, education, and the distribution of income. Am Econ Rev 65:283–300
Stokey NL, Rebelo S (1995) Growth effects of flat-rate taxes. J Polit Econ 103:519–550
Stutzer M (1980) Chaotic dynamics and bifurcation in a macro economics. J Econ Dyn Control 2:253–273
Tournemaine F, Luangaram P (2012) R&D, human capital, fertility, and growth. J Popul Econ 25:923–953
Uzawa H (1965) Optimal technical change in an aggregative model of economic growth. Int Econ Rev 6:18–31
Varvarigos D, Zakaria IZ (2013) Endogenous fertility in a growth model with public and private health expenditures. J Popul Econ 26:67–85
Wang P, Yip CK, Scotese CA (1994) Fertility choice and economic growth: theory and evidence. Rev Econ Stat 71:255–266
Wigger BU (1999) Pay-as-you-go financed public pensions in model of endogenous growth and fertility. J Popul Econ 12:625–640
Yip C, Zhang J (1997) A simple endogenous growth model with endogenous fertility: indeterminacy and uniqueness. J Popul Econ 10:97–100
Zhang WB (1993) Woman's labor participation and economic growth: creativity, knowledge utilization

and family preference. Econ Lett 42:105–110
Zhang J (2003) Optimal debt, endogenous fertility, and human capital externalities in a model with altruistic bequests. J Popul Econ 87:1825–1835
Zhang WB (2005) Economic growth theory. Ashgate, London
Zhang WB (2012) Education and human capital accumulation in a two-sector growth model with elastic labor supply. Annals of the Alexandru Ioan Cuza University of Iasi, Economic Sciences Section 2012:289–309